Project Internship Report

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Contents

1 Abstract

This report deals with the development of a translation from the synaptic weight in hardware of the LIF neurons used in the BrainscaleS-1 waferscale system to the biological weight used in simulation. Since the only observable of these neurons is the membrane voltage we looked at the voltage course after a synaptic spike. We then looked at different functions to approximate the synaptic weight from the height of the Post synaptic potential (PSP). We found that the approximation will not be accurate enough for a calibration routine which is why we looked at different functions to describe the voltage course and reconstruct the synaptic weight. In a first hardware measurement we saw some additional saturation behaviour which still needs to be explained by simulating the hardware and looking at non ideal effects in different components of the neuron circuit. In the future we will look for a good set of parameters to minimize this impact.

2 Introduction

The leaky integrate and fire model (LIF) is used to describe the behaviour of neurons. For conductance based neurons this model can be described by the following differential equation.

$$c_m \frac{dV}{dt} = -g_l * (V - V_l) + \sum_{i=1}^{K} g_i^{syn}(t) * (V - V_i) \tag{1}$$

This formula describes the membrane voltage over the membrane capacity c_m. In the case of no synaptic input the voltage approaches the leakage (or resting) voltage V_l due to a currentflow over the leak conductance g_l. Additionally an extra term would describe the process of the neuron firing in case the voltage crosses a threshold voltage. This term is neglected in this report as we operate below the firing threshold. In case of synaptic input the conductance g_i of the corresponding synapse connection changes. The synaptic conductance is given by:

$$g_i^{syn}(t) = w * exp(-\frac{t}{\tau_{syn}}) \tag{2}$$

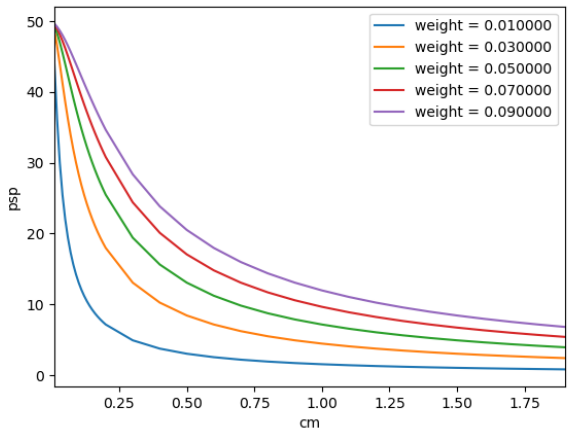
The decay of the synaptic conductance after the arrival of a spike is described by the synaptic time constant τ_{syn}. At t=0 the synaptic conductance is given by the synaptic weight w. The change of the conductance leads to a current due to the difference of the membrane voltage and the synaptic reversal potential V_i. The reversal potential will later be called *E_rev_E* for excitatory reversal potential, since we will only look at the voltage course after the arrival of a excitatory spike. This differential equation can not be solved analytically but it can be calculated numerically, though this is slow and not energy efficient. For this reason the electronic vision(s) group takes a different approach. By building a electronic circuit which is described by the same differential equation, the evolution of the circuit solves the equation implicitly. A lot of these circuits are implemented on the waferscale system BrainScaleS-1. Its electronic circuits represent about 4 million neurons and around one billion synapses.
When doing research a lot has to be done using simulation. For example to verify the correctness of the

measurements. In order to simulate measured data, the parameters on hardware have to be translated into the biological parameters used in the simulation. This project describes the steps to find a translation of the synaptic weights of each neuron on hardware to those used in the simulation. Due to differences between neurons, e.g. in the doping strength, a weight calibration is necessary to make the comparison of simulation and hardware for each individual neuron possible. In order to do so we look at the effect of single synaptic spikes on the membrane potential of the neurons and analyse the resulting postsynaptic potentials.

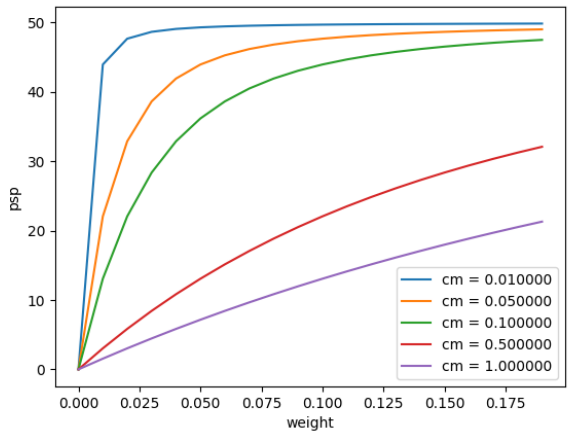
3 Dependencies of the Synaptic Weight

We first look at the dependencies of the synaptic weight in simulation. Here it is important to note, that the synaptic weight is a parameter of the synapse connection. The postsynaptic Potential (PSP) is therefore the result of a synaptic spike with the according synaptic weight.
Using the simulator NEST for spiking neural networks we are able to investigate the course of the PSP depending on the neuron parameters. To identify which neuron parameters were important we varied the parameters with some others fixed.
In the following plots we only look at the height of the PSP. This height is described by the maximum of the membrane voltage and the resting potential (the voltage of the membrane wihout any spikes). In Figure 1 we can see that the curves of Figure 1a are monotonously falling and approach 0 for higher c_m. In Figure 1b we see the complementary effect. The height approaches a maximum for higher weights. A higher membrane capacitance leads to a slower approach of this maximum. The simplest function to have this behaviour would be the height being reciprocally dependent on the membrane capacity, though this would diverge for c_m to 0. We therefore also expect a saturation of the PSP as the height seems to approach a maximum of 50 with the here used neuron parameters. With the conductance based neuron model a saturation of the height makes sense, as the maximum membrane potential is the excitatory reversal potential.
We next look at the dependence of the PSP height on the resting and the excitatory reversal potential. We find opposite behaviours of the course of the PSP height for both parameters. For the reversal potential in Figure 2 we can see what appears to be a linear rise between the PSP height and the reversal potential. In Figure 3 we find a linear decrease of the PSP height with the resting potential. We therefore expect a linear dependence of the PSP height on the difference between the resting and reversal potential.

The relation of the PSP height as a function of the membrane time constant is not easily identified from the graphs. From the course we suspect an exponential relation. What exactly this relation looks like we do not know however. The same is true for the synaptic time constant. We see a very similair course though the PSP height seems to saturate faster in Figure 5a compared to Figure 4a. We are however unable to conclude anything more.
The height of the PSP therefore depends on the difference between excitatory synaptic potential and the leakage potential, an unknown dependence on the time constants τ_m and τ_{syn_E} , reciprocally on the membrane capacity c_m and the synaptic weight. We also expect a saturation of the PSP for a constant

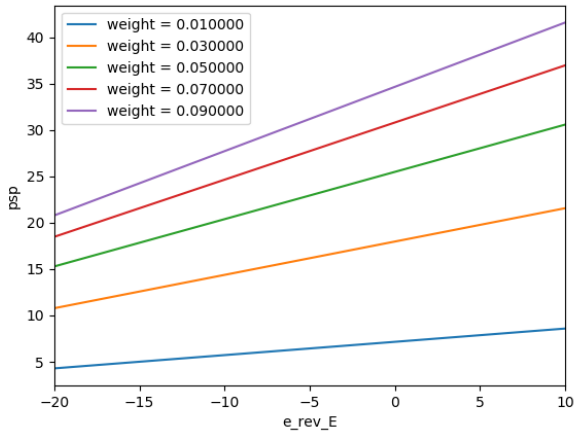


(a) PSP over c_m with different weights

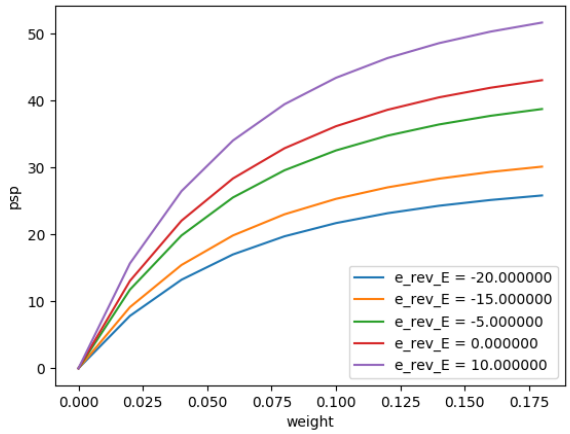


(b) PSP over weights with different c_m

Figure 1: PSP dependence on the neuron parameter c_m



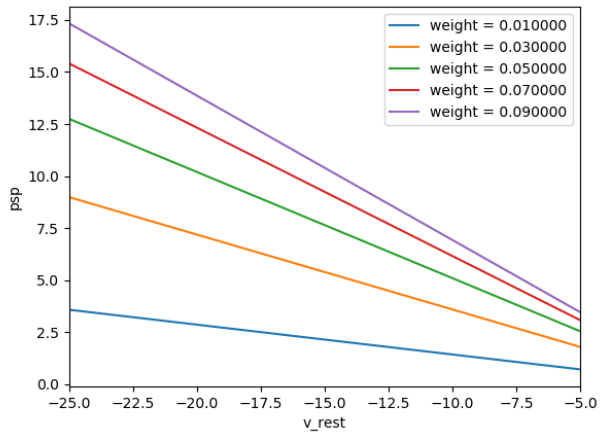
(a) PSP over E_rev_E with different weights



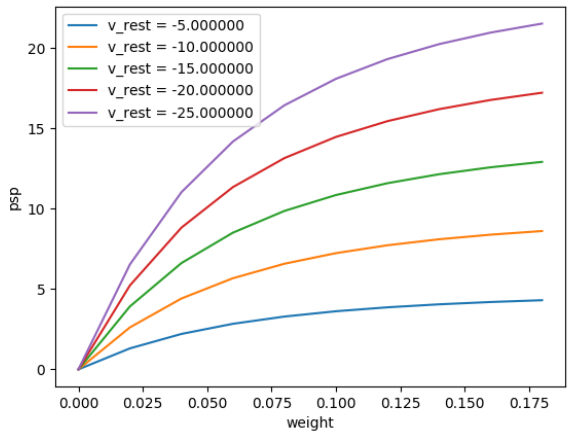
(b) PSP over weights with different E_rev_E

Figure 2: PSP dependence on neuron parameter E_rev_E

difference between E_rev_E and V_{rest}. Since we later want to find the weight depending on the height we will first look for a linear approximation of the dependency as this would be the easiest to invert and therefore fastest to implement in a calibration algorithm on the hardware. This dependency is however greatly dependent on the neuron parameters. We are also not able to change the membrane capacity on the hardware. Our calculations later on will therefore use $\frac{w}{c_m}$ instead of just w. In order to compare the from hardware measurements calculated weight, one just needs to set the membrane capacity to 1 in the simulation. In Figure 6 we see two postsynaptic potentials with the same neuron parameters but different weights. The rise of the voltage to the left comes from the initialization of the simulation. The voltage starts at a on the simulation dependent value and then approaches the resting potential due to the

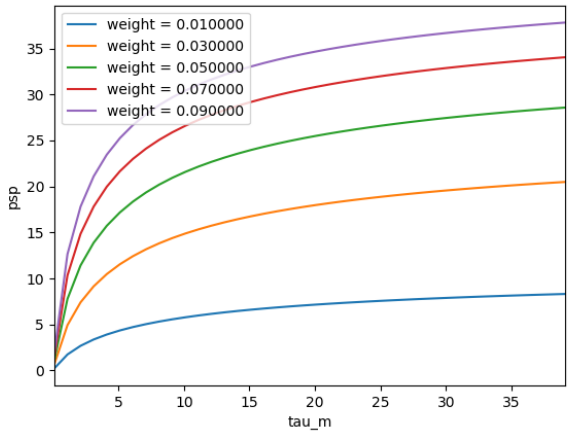


(a) PSP over V_{rest} with different weights

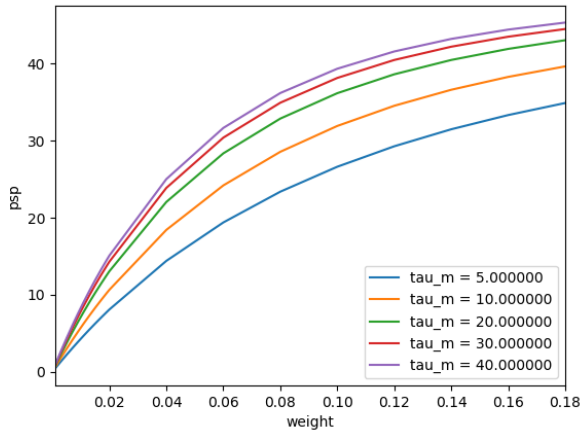


(b) PSP over weights with different V_{rest}

Figure 3: PSP dependence on neuron parameter V_{rest}



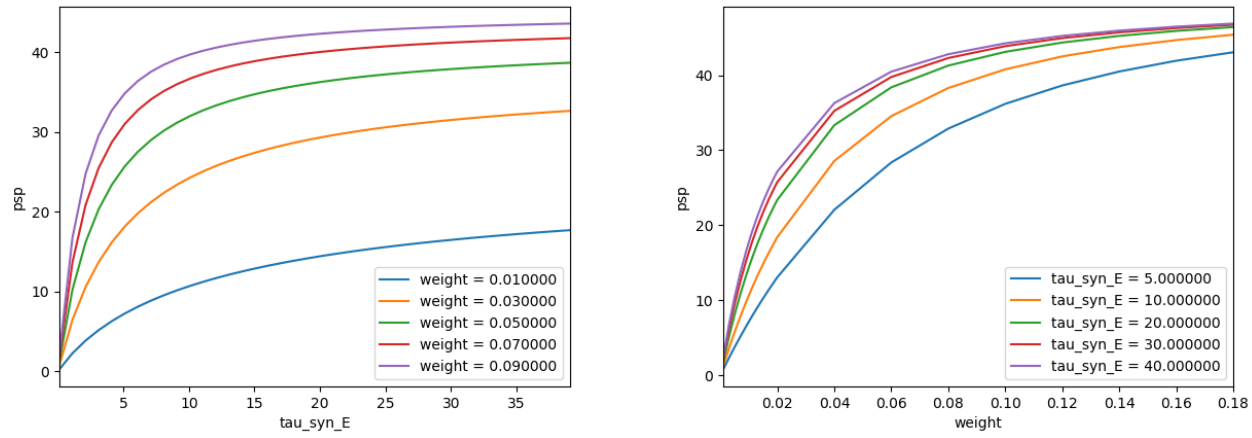
(a) PSP over τ_m with different weights



(b) PSP over weights with different τ_m

Figure 4: PSP dependence on neuron parameter τ_m

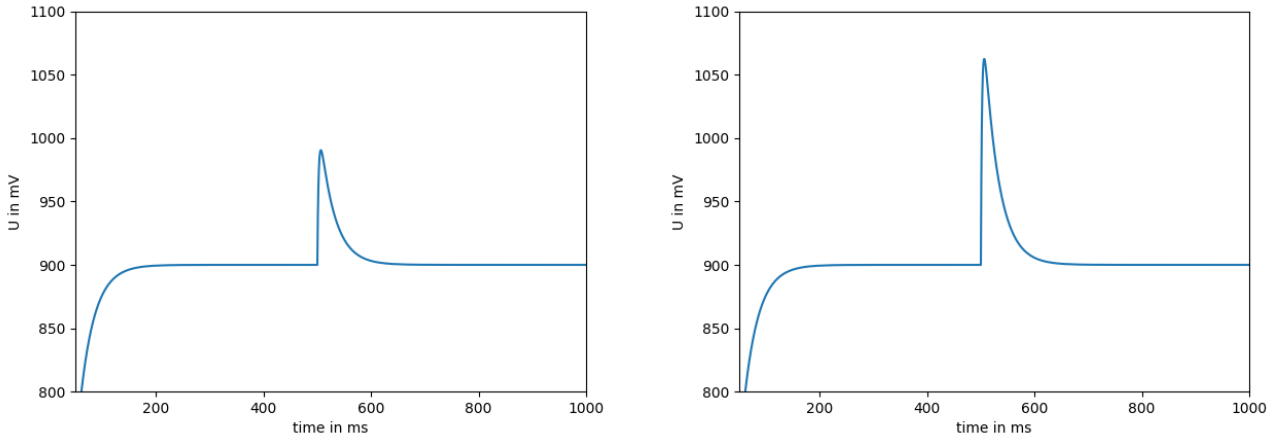
leak current. At a time t_0, here 500ms, a synaptic spike leads to a peak on the membrane potential. This is the postsynaptic potential or PSP. We also see that the height is not linear in the weight. As the weight increases from Figure 6a to Figure 6b by a factor of 2 the height increases only by a factor of about 1.7. The reason for this is a saturation effect as the potential approaches the reversal potential. The current over the conductance decreases as there is a higher voltage drop over the membrane condensator. The maximal height of the PSP is therefore given by the difference of the reversal and leakage potential. The form of the PSP follows the solution of Equation 1.



(a) PSP over τ_{syn} with different weights

(b) PSP over weights with different τ_{syn}

Figure 5: PSP dependence on neuron parameter τ_{syn}



(a) PSP with a weight of 0.1

(b) PSP with a weight of 0.2

Figure 6: Two PSP at different weight settings

4 Approximation of the PSP Height and Synaptic Weight

The dissertation of Christoph Koke investigates the effects of a single spike on the membrane potential of a neuron [1]. In his work he uses a linear approximation of the PSP height depending on the synaptic weight for neurons in the high conductance state by finding the maximum of the membrane voltage course given by

$$V(t) \approx V_{rest} + \Theta(t_0) * A * (exp\left(\frac{t_0 - t}{\tau_m}\right) - exp(\left(\frac{t_0 - t}{\tau_{syn}}\right)). \tag{3}$$

This voltage course claims to approximate the conductance based equation of Equation 1 with the condition of a single spike input and the voltage being in a steady state at the arrival of the spike. By finding the time of maximum of the PSP and inputting the result in the voltage course we get the height of the PSP given by

$$h = A * (\tau^{\frac{\tau}{1-\tau}} - \tau^{\frac{1}{1-\tau}}) \tag{4}$$

here $\tau = \frac{\tau_{syn}}{\tau_m}$ and

$$A = \frac{w * (E\_rev\_E - V_{rest}) * \tau_g}{c_m} \tag{5}$$

with w the synaptic weight and $\tau_g = (\frac{1}{\tau_{syn}} - \frac{1}{\tau_m})^{-1}$. Also τ_{syn} and τ_m are the synaptic and membrane time constants respectively (measured in ms), c_m is the membrane capacity (in nF). E_rev_E and V_{rest} are the excitatory reversal potential and the resting potential (measured in mV).
Due to the linear relation between height and synaptic weight it is now easy to calculate the dependence of the weight on the neuron parameters and the height of the PSP. This leads to the following equation which describes the weight for a neuron with the previously written criteria:

$$w = \frac{h * c_m}{(E\_rev\_E - V_{rest}) * \tau_g * (\tau^{\frac{\tau}{1-\tau}} - \tau^{\frac{1}{1-\tau}})} \tag{6}$$

However on hardware it is not possible to directly calculate the applied synaptic weight, since we are not able to accurately set the neuron parameters. The parameters are saved in floating gate cells, the resulting currents or voltages then set the corresponding neuron parameters. The resulting time constants τ_i for example are distributed with a standard deviation σ of 20% their mean, similarly the difference of the voltages is distributed with a σ of 10% its mean. Since these errors are of a statistical nature we are not able to correct them. The total error can be calculated with Gaussian error addition. However we only take the first order into account here. This would lead to errors in the order of 20%, depending on the neuron parameters.
For this reason we will look at the average over multiple neurons. At the time of writing this report the averaging of the neurons was not implemented yet. The theoretical results of averaging are shown in the next paragraph.

5 Average of neurons

The average of a function f dependent on x is defined as

$$\overline{f} = \int_{-\infty}^{+\infty} f(x)\rho(x)dx \tag{7}$$

with $\rho(x)$ being the normed density function. For a function depending on a parameter x that is Gaussian distributed we can therefore calculate the mean of the function with the $\rho(x)$ being a normed Gaussian. Since we want to calculate the mean of the synaptic weight which is a parameter in our measurement of the PSP height our average looks like this (with the function of h taken from Equation 4 and Equation 5):

$$\overline{h} = \int_{-\infty}^{+\infty} h(w, E\_rev\_E, ...) * \rho(w) * ... * \rho(E\_rev\_E) dw...dE\_rev\_E \tag{8}$$

This integral separates which is why we can pull out the average of $\frac{w}{c_m}$. This is the quantity we want to calculate. We therefore get:

$$\overline{\left(\frac{w}{c_m}\right)} = \frac{\overline{h}}{\overline{(E\_rev\_E - V_{rest})} * \int_{-\infty}^{+\infty} f(\tau_m, \tau_{syn}) d\tau_m d\tau_{syn}} \tag{9}$$

With $f(\tau_m, \tau_{syn})$ being the function depending on the τ_i from Equation 6. Since $f(\tau_m, \tau_{syn})$ does not depend linearly on the τ_i the integral in the fraction will have a different value than the value of the function with the averaged τ_i. An example of this can also be seen in Figure 7. The Gaussian distributions of the time constants does not lead to a symmetric shape of the weight distribution. The mean will therefore differ from the weight calculated with the averaged neuron parameters. The shown weight distribution was calculated with the same neuron parameters as the ones in Figure 6 with errors on the time constant of 20%. Since this error is systematic and fixed for chosen means of neuron parameters, it can however be corrected after the measurement. With this a calculation of an averaged weight for many neurons is possible. It is important to note, that an additional statistical error arises when the average is taken

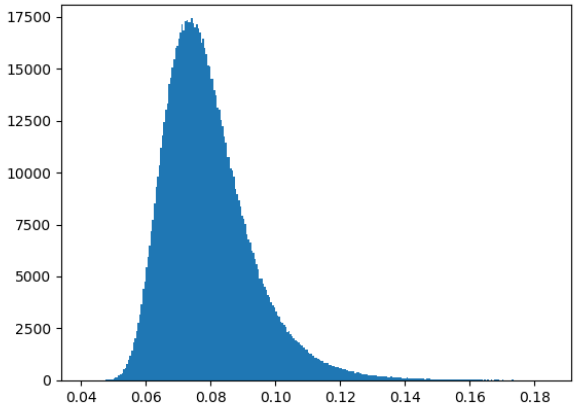


Figure 7: Weight distribution with Gaussian distributions of the time constants

over a limited number of neurons. The average will therefore not have the value calculated here but

differ between measurements of different neuron groups as the density distribution only converges to a Gaussian for a high number of neurons. This error however can be estimated depending on the number of neurons which are averaged and is much smaller than the stochastic error of a single measurement. For different ways to calculate the synaptic weight from the height of the PSP this distribution changes. As we already saw in e.g Figure 1 the height will saturate, meaning that the linear approximation does not describe the PSP for higher weights. Another relation is therefore needed. When we determine the final calibration routine we will need to see how an averaging of neurons would impact the measurement.

6 Hardware

We now look at the hardware. Unfortunately we can not adjust the neuron parameters as easy as in the simulation. Especially the time constants are difficult to set up. The HICANN uses control currents and voltages that then change resistive elements to set these parameters. Problematic is, that we are not able to measure these as easily, which is why we do not know the exact parameters for our calibration. However it is possible to measure the membrane voltage after the synaptic spike. Our measurement program then measures the PSP and fits the function from [1, Equation 2.13] on the course of the membrane voltage. The fitted height can be seen in Figure 8. We can already see a saturation taking place. We also find, that for low weights no PSP will be recorded. The reason for this is the voltage $V\_convoff$ which cuts of too small membrane voltages in order to avoid negative currents in the synaptic input circuit. With the fit the time constants will also be fitted. We find that the time constants do not appear

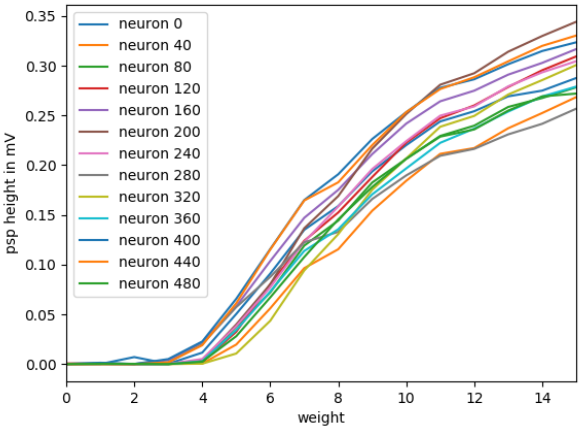
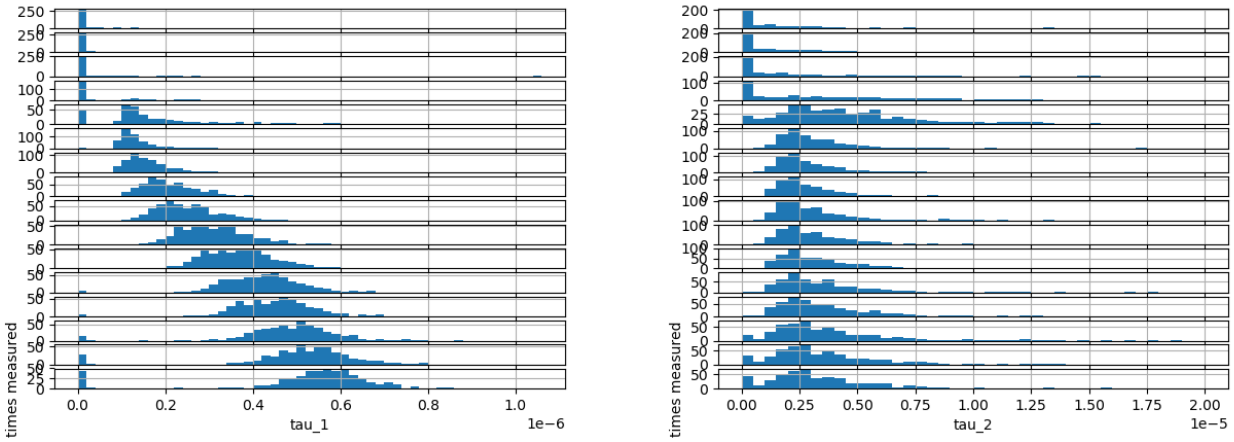


Figure 8: Height of the PSPs with different weights

to be constant for different weight settings (see Figure 9). This can partly be explained by the resistive element that leads to the synaptic time constant. Ideally its resistance would only depend on the bias voltage V_syntc which is used to set the time constant. However the resistance also depends on the voltages at its terminals. For higher weights this voltage increases which would lead to a shift in the time constants. We will need to look in simulation if this explains the shift seen in Figure 9a or if the

form of the PSP changes as the fitted model does no longer describe the voltage course.



(a) Fitted synaptic time constant τ_{syn}

(b) Fitted membrane time constant τ_m

Figure 9: Histograms of fitted time constants, weight increases from top to bottom

We see a rise in the synaptic time constant and a slight decrease in the membrane time constant. We also again find that the PSPs are not produced for small weights, for this reason the time constants can not be fitted. We also see a similar trend for high weights, which can not be explained at this moment but might happen due to a change in the form of the PSP which does not allow the fit to conclude successfully. More research to understand this behaviour will be necessary. Apart from these we also use the measured PSP to fit the leakage potential. This potential is constant for the different weights. This could be expected as it does not depend on the form of the PSP. For CUBA and COBA neurons the potential starts at the leakage potential and approaches the leakage potential again after the peak.

After measuring the PSPs for 512 neurons we then use Equation 6 to calculate the according weight in biological parameters using the fitted neuron parameters and the height of the measured PSP. To check our results we can then again use the NEST simulation. After inputting the neuron parameters and the calculated weight the PSP should be the same as the before measured one if the weight was calculated successfully. We find that the simulated PSPs are smaller than the measured ones. This could be expected as the PSP height saturates with higher weights. A high input weight will therefore be translated to a smaller biological weight as the PSP height in the linear approximation appears to come from a smaller input weight. Since the linear approximation is only accurate for smaller weights we find that the discrepancy between the measured height and the simulated height rises with the weight.

If we do not use the fitted values but instead the values that were chosen as the neuron parameters, we find that the discrepancy only slightly increases. The from the height calculated weight also rises more smoothly than with the fitted parameters.

Reasons for the discrepancy between the simulated/measured PSP height and the linear approximation

come from the difference of conductance and current based neurons. The neurons used are conductance based. The current therefore depends on the difference between the membrane potential and the excitatory reversal potential, leading to a saturation of the voltages.

7 Change in the Height Model

In order to accommodate the saturation in our model we change the model function for some fast approximation of the weight depending on the height. We know that the linear approximation from Equation 4 describes our data well for very small weights. The first order Taylor expansion of the new model should therefore be identical to linear approximation. We also know that we charge a capacitance over a conductance with applied voltages of the resting potential and the excitatory reversal potential. The maximum voltage of the membrane (i.a. the capacity) is for this reason the difference between the potentials. This is the maximum height of the PSP. For the new model we now choose a limited growth function. The criteria of the maximum and the first order Taylor term fully define this function.

$$h = h_{max} * (1 - e^{-\kappa * w}) \tag{10}$$

From out two requirements we find

$$h_{max} = E\_rev\_E - V_{rest} \tag{11}$$

and

$$\kappa * h_{max} = \frac{\tau_g * (E\_rev\_E - V_{rest})}{c_m} * (\tau^{\frac{\tau}{1-\tau}} - \tau^{\frac{1}{1-\tau}}) \tag{12}$$

Using Equation 10 we get our new model for the PSP height depending on the weight.
When looking at the calculated PSP height we find a discrepancy 8% to 19% depending on the neuron parameters toward the simulated PSP height. We also find that the PSP is not symmetric in the time constants unlike in our approximation. This has a grand impact as the maximal discrepancy happened with a much larger τ_{syn} then τ_m and in contrast the minimal discrepancy with a much larger τ_m then τ_{syn}. To achieve a even better result we use the time constants measured from the fit of the PSP. The minimal discrepancy does not change by a lot. However the maximal discrepancy changes drastically as the usage of the fit seems to eliminate the error from a in the τ_i non symmetric simulation. For the looked at neuron parameters we only found a maximum error of 10%.
To reconstruct the weight from the measured PSP height we invert Equation 10. This leads to

$$w = \frac{ln(1 - \frac{h}{h_{max}})}{-\kappa} \tag{13}$$

Since this function diverges as the height approaches the difference of the resting and reversal potential h_{max}, the small discrepancy of the simulated and with Equation 10 calculated PSP height leads to a

larger discrepancy in the weight. Another problem is that we reconstruct the height of the PSP during the hardware measurement by fitting Equation 3 on the course of the membrane voltage. However the form of the PSP changes for higher weights which is why the height of the PSP is chosen so that the overall function describes the data best. An example of the fit can be seen in Figure 10. As can be seen the height from the fit overestimates the actual height of the peak. In case that the fitted height is higher then the difference of the potentials Equation 13 will lead to no result as the logarithm of a negative number is only complex. We know that this formula does not accurately describe the relation of PSP height and weight. However we later use this estimation to find a good starting point for a fit on the PSP as it is more accurate but as fast as the linear approximation used before.

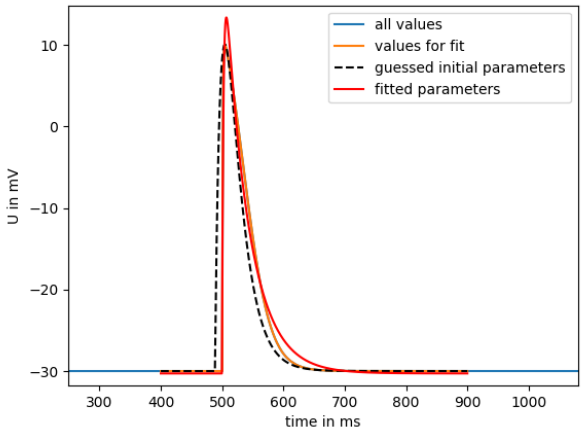


Figure 10: Fit on the membrane voltage

8 Analytical Solution of a COBA neuron

Since our approximations did not provide us with the intended accuracy we again look at Equation 1. We find in [2, equation B.5] a solution of the differential equation which proved to describe the simulated PSP. However the solution contains a gamma function which can only be calculated numerically. The solution looks as follows:

$$\begin{aligned} V(t) =& exp(-\frac{t}{\tau_m^l} + \frac{\tau_s}{\Delta\tau_m^s} e^{-\frac{t}{\tau_s}}) * \Big(E_l e^{-\frac{\tau_s}{\Delta\tau_m^s}} - E_s (e^{-\frac{\tau_s}{\Delta\tau_s^m}} - exp[\frac{t}{\tau_m^l} - \frac{\tau_s}{\Delta\tau_m^s} e^{-\frac{t}{\tau_s}}]) \\ & - \Gamma[-\frac{\tau_s}{\tau_m^l}, \frac{\tau_s}{\Delta\tau_m^s} e^{-\frac{t}{\tau_s}}, \frac{\tau_s}{\Delta\tau_m^s}] (\frac{\tau_s}{\Delta\tau_m^s})^{\frac{\tau_s}{\tau_m^l}} \frac{\tau_s}{\tau_m^l} (E_s - E_l) \Big) \end{aligned} \tag{14}$$

The notations in [2] are a little different then those previously used. τ_m^l is the membrane time constant τ_m, τ_s is the synaptic time constant τ_{syn} and $\Delta\tau_m^s$ is the quotient of membrane capacity and synaptic weight $\frac{c_m}{w}$. We again find that the weight only appears in connection with the membrane capacity, this

again motivates us to conduct the calibration not just of the weight but of the quotient $\frac{w}{c_m}$.
The Gamma function with three arguments is defined by:

$$\Gamma[z,a,b] = \int_a^b t^{z-1} e^{-t} dt \tag{15}$$

Because this equation contains the synaptic weight, a fit of this function on the PSP allows us to find the parameter we are looking for without needing to use the height of the PSP. However since the gamma function can not be solved analytically even a plotting to compare to the PSP takes a lot of time (order of seconds). A fit routine evaluating the function a lot more often is therefore not suitable for calibration due to the very large number of neurons per chip and multiple measurements being necessary for good results. For this reason we look into ways to speed up the fit.
One way we found was to let the scipy module odeint solve the differential equation. This sped up the process by two orders of magnitude so that a fit could be conducted in about one second. The duration of the fit heavily depends on the start parameters given. Using Equation 13 proved to give good enough approximations for the fit to work consistently. We will need to check how much the noise on the voltage on hardware will impact the fit quality and its duration. For this reason some more tests with on hardware measured PSPs are necessary.

9 Outlook

In order to calibrate the synaptic weight in hardware and biology for each individual neuron the next steps will be to do more measurements on hardware. During the time of the Internship a first measurement run was conducted. However the results obtained were only used in connection with the linear approximation and the fit of Equation 3. In the future the results will be used with the solution to Equation 1 to see whether the PSP measured can be reproduced with the fitted weight in simulation. A challenge with this will be to only use those neurons which produced clear PSPs. Some noise on the PSP which should be fitted already greatly increased he duration of the fit and sometimes led to a wrong estimation of the parameters. During the first measurement it became apparent that the Fit did not work for small PSPs at all due to a cutoff on the hardware. For this reason neuron parameters which produce high peaks might be useful, though we need to see if we would then get in a regime where saturation effects also impact the PSP. The measurement therefore needs to be conducted with different setup neurons. We also need to look at the quality of the fitted time constants. During the first measurements we saw a linear trend in the time constants which could also be found in the simulation. We will have to see whether this just came from the wrong model or whether the hardware resistor changes depending on the used voltages. Also there were a lot of outliers which produced PSPs which could not be explained by our approximations. We need to see if this is the fault of the neurons, by averaging over more measurement runs or if another problem leads to this behaviour. We will also have to see whether the saturation happening on hardware is just the result of the neuron model. On hardware there however might be additional effects which lead to a saturation, e.g. limited currents. We also

need to be sure of the exact value of the voltages. During the first measurement we found a constant leakage potential, with a working fit we will have to see if the excitatory reversal potential behaves also as expected.

References

[1] Christoph Koke, *Device Variability in Synapses of Neuromorphic Circuits*, Heidelberg, 2017.

[2] Michelle Rudolph, Alain Destexhe *Analytical Integrate-and-Fire Neuron Models with Conductance-Based Dynamics for Event-Driven Simulation Strategies*, Massachusetts Institute of Technology, 2006.